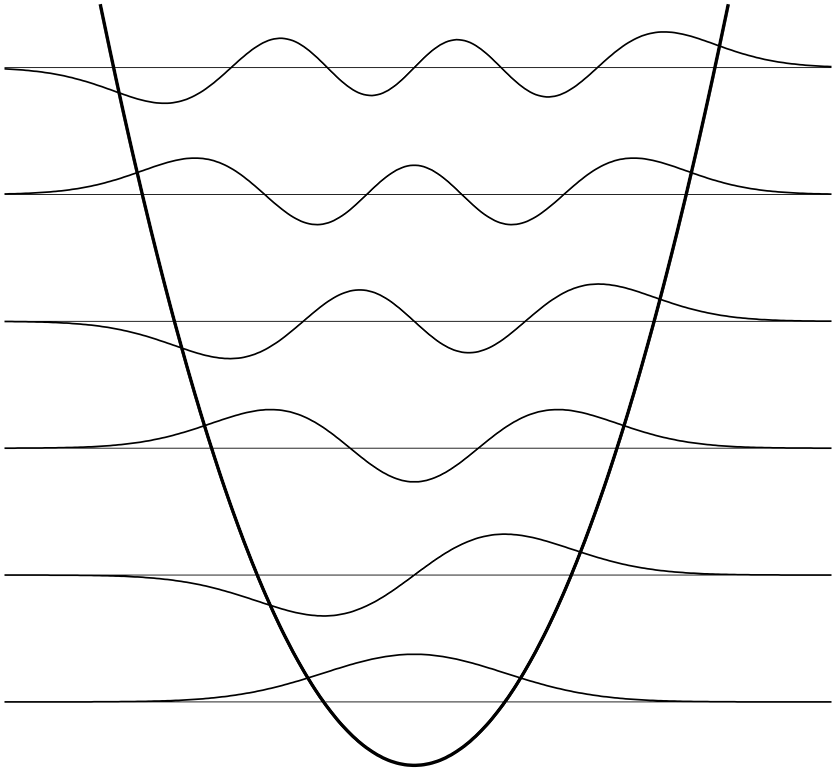
harmonic oscillator



$$-\frac{1}{2}\frac{d^2\varphi}{d\zeta^2}+\frac{\zeta^2}{2}\varphi=\varepsilon\varphi$$

$$\varphi_0(\zeta)=a_0e^{-\zeta^2/2} \qquad \varepsilon_0=1/2$$

$$a\;=\frac{1}{\sqrt{2}}\left(\zeta+\frac{d}{d\zeta}\right)$$

$$a^\dagger=\frac{1}{\sqrt{2}}\left(\zeta-\frac{d}{d\zeta}\right)$$

$$\left(a^\dagger a+1/2\right)\varphi=-\frac{1}{2}\frac{d^2\varphi}{d\zeta^2}+\frac{\zeta^2}{2}\varphi=\varepsilon\varphi$$

recursively construct eigenstates

$$aa^\dagger=a^\dagger a+1$$

$$\left(a^\dagger a+1/2\right)\;a^\dagger\varphi_0=3/2a^\dagger\varphi_0$$